Math 134—Managerial Calculus
Measurable Outcomes

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Reference text: Numbers in brackets refer to sections of Tan, *Applied Mathematics for the Managerial, Life, and Social Sciences,* seventh edition.

Note: Outcomes marked **(Optional)** may appear on the final exam with the unanimous consent of all instructors.

1. Limits

- **1(a)** Find limits by the method of direct substitution [9.1]
- **1(b)** Find limits for indeterminate forms of type 0/0 by methods of factoring and rationalizing numerators. [9.1]
- **1(c)** Recognize that a limit does not exist when it exhibits unequal one-sided behaviors.[9.1]
- **1(d)** Correctly use the infinity symbol where it is appropriate. [9.1]
- **1(e)** Find limits at infinity of rational functions. [9.1]
- **1(f)** Find intervals where functions are continuous or values where functions are discontinuous. [9.2]
- **1(g)** **(Optional)** Using the Intermediate Value Theorem, verify that roots of a polynomial exist in a given interval. [9.2]

2. The Derivative

- **2(a)** Define derivative as a limit. [9.3]
- **2(b)** Find a derivative from its definition via the Four Step Process. [9.3]
- **2(c)** Use differentiation formulae: sum and difference rules, constant multiple rule, product rule, quotient rule. [9.4, 9.5]
- **2(d)** Calculate higher-order derivatives. [9.5]
- **2(e)** Find an equation of the tangent line to a curve at a specified point [9.3, 9.4, 9.5]
- **2(f)** Recognize when a curve fails to have a tangent line at a point.[9.3]

2(g) Use the Chain Rule to differentiate composite functions. [9.6]

2(h) Combine the Chain Rule with the product and quotient rules when appropriate. [9.6]

3. Differentiation of Exponential and Logarithmic Functions

3(a) Differentiate the natural exponential and logarithmic functions. [9.7]

3(b) **(Optional)** Differentiate the general exponential function a^x and logarithmic function $\log_a(x)$, when $a > 0, a \neq 0$.

3(c) Combine exponential and logarithm rules with the Chain Rule, the Product Rule, and the Quotient Rule. [9.7]

3(d) Calculate higher-order derivatives involving exponential or logarithmic functions. [9.7]

3(e) Find vertical and horizontal asymptotes for functions involving logarithms or exponentials. [9.7]

4. Marginal Functions in Economics

4(a) Demonstrate that the actual cost of production of the nth unit (determined by computing the difference of costs), is closely approximated by the marginal cost (the derivative of the cost function). [9.8]

4(b) Find marginal cost function, average cost function, marginal average cost function, marginal revenue function, and marginal profit function. [9.8]

4(c) Identify vertical and horizontal asymptotes for average cost function in linear cost model, and give their economic interpretations. [9.8]

4(d) Find absolute, relative and percentage rates of change, e.g. for the consumer price index. [9.8]

5. Applications of the Derivative to Analysis and Graphing

5(a) Use the first derivative for determining the shape of the graph: calculate intervals of increase/decrease, local extrema, and critical numbers, and tabulate these results in an orderly fashion. [10.1]

5(b) Use the second derivative for determining the shape of the graph: calculate intervals of concavity and points of inflection use the Second Derivative Test for local extrema, and tabulate the results in an orderly fashion. [10.2]

5(c) Sketch curves via an orderly process of steps using domain, limits, asymptotes, first and second derivatives [10.3]

5(d) Use the graph to determine range of the function. [10.3]

6. Optimization

6(a) Find the absolute maximum and minimum values of a continuous function by use of the Closed Interval Method. [10.4]

6(b) Find the absolute maximum and minimum values of a function where the function has a discontinuity, or the interval is open. [10.4]

6(c) Use the Second Derivative Test to identify absolute extrema on open intervals. [10.4]

6(d) Apply the optimization techniques described above to word problems, where the objective function is not supplied but deduced from the wording. [10.5]

6(e) Find domain of the objective function of a word problem by careful reading of physical circumstances of the problem. [10.5]

7. Indefinite Integration

7(a) Demonstrate that antidifferentiation is the reverse of differentiation by taking the derivative of an antiderivative to get the original function. [11.1]

7(b) Demonstrate understanding of basic rules of integration by examples: constant, Power Rule, constant multiple of a function, sum of functions, natural exponential function, reciprocal function. [11.1].

7(c) **(Optional)** Integrate the general exponential function.

7(d) Solve initial value problems for differential equations. [11.1]

7(e) Calculate indefinite integrals by the method of substitution [11.2]

8. Definite Integration

8(a) **(Optional)** Approximation areas by Riemann sums. [11.3]

8(b) Interpret definite integrals as signed areas. [11.3]

8(c) Use the Fundamental Theorem of Calculus to evaluate definite integrals. [11.4]

8(d) Use the Fundamental Theorem of Calculus to find the area of a region under a curve, in a given interval where the function is positive. [11.4]

8(e) Calculate definite integrals by the method of substitution. [11.5]

8(f) Calculate the area between two curves and bounded by two vertical lines. [11.6]

8(g) Calculate the area enclosed by two curves. [11.6]

8(h) Use definite integration formulas to find consumers' surplus. [11.7]

8(i) Use definite integration formulas to find producers' surplus. [11.7]

8(j) Use definite integration formulas to find the future value of an income stream. [11.7]

8(k) Use definite integration formulas to find the present value of an income stream. [11.7]

9. Introduction to Multivariable Calculus

9(a) **(Optional)** Find and graph in the plane the domain of a function of two variables. [12.1]

9(b) **(Optional)** Evaluate functions of several variables. [12.2]

9(c) **(Optional)** Calculate the partial derivatives of a function of several variables. [12.2]

9(d) **(Optional)** Verify by direct calculation of examples that when a function has continuous mixed second partials, its mixed second partials are equal. [12.2]